Universität zu Köln
Markets for Risk Management

Problem Set #2 Solutions
3 June 2013
Professor Garven

Problem #1 (33 points)

Suppose a share of stock currently trades for €50. A call option written on this stock with an exercise price of €50 trades for €2, and an otherwise identical put option also trades for €2. The options are both European and expire 1 month from today.

1. (11 points) Describe a trading strategy involving the call, the put, and the share that enables you to synthetically replicate a riskless pure discount bond[1] with a face value equal to the exercise price on these options.

   SOLUTION: According to the put-call parity theorem, one can synthetically replicate a riskless pure discount bond with a face value equal to the exercise price on these options by purchasing the put, buying the share, and selling the call; assuming that $\delta t = 1/12$, then

$$c + \frac{X}{(1 + r\delta t)} = p + S \Rightarrow \frac{X}{(1 + r\delta t)} = p + S - c.$$

2. (11 points) What is annualized riskless rate of interest implied by the prices of these various securities?

   SOLUTION: Since $\frac{X}{(1 + r\delta t)} = p + S - c$, this implies that $X = (1 + r\delta t)(p + S - c)$; Thus, $50 = (1 + r(.083))(2 + 50 - 2) \Rightarrow r = 0$.

3. (11 points) Suppose that the annualized riskless rate of interest is 5%. Describe an arbitrage strategy that will enable you to make riskless profits with zero net investment. Calculate the profits that are earned, and also numerically confirm that the profits are riskless and do not involve any investment of your own money.

   SOLUTION: Since the annualized riskless rate of interest is 5%, this implies that compared with the synthetic bond, a real bond is cheap. Specifically, the synthetic bond costs €50, whereas a real bond that matures in one month costs $\frac{50}{(1 + .05(.083))}$ = €49.79. Therefore, the obvious thing to do is to sell the synthetic bond for €50, buy the real bond for €49.79, and pocket the difference of €50-€49.79 = €0.21 as profit. One month from now, the value of the hedge portfolio (V_1) will be €0 no matter what happens to the values of the options and the shares. Note that $V_1 = -p_1 - S_1 + c_1 + X = -Max(X - S_1, 0) - S_1 + Max(S_1 - X, 0) + X$. If $S_1 \geq X, V_1 = 0 - S_1 + S_1 - X + X =$ €0. On the other hand, if $S_1 < X$, then $V_1 = -X + S_1 - S_1 + 0 + X =$ €0. This is a perfectly hedged position, zero net investment strategy which generates up-front profit of €0.21.

[1] A pure discount bond is a bond that only pays interest (as well as principal) on the date of maturity.

Problem #2 (32 points)

Suppose you are interested in pricing a European put option which matures in 1 year. Currently, the underlying stock for this option is worth €40, and its exercise price is €40. Assume that the (annualized) riskless rate of interest is 5 percent, and that the (annualized) volatility of the stock is 35 percent. Here are some other facts that are important in valuing this option:

- Assume that the "up" move, $u = 1 + \sigma\sqrt{\delta t}$, and the "down" move $d = 1 - \sigma\sqrt{\delta t}$;
- Assume discrete discounting; therefore the present value discount factor ($PVDF$) for an interest rate of r and one time step is $PVDF = 1/(1 + r\delta t)$.

1. (8 points) Using a two-period binomial tree, solve for the current market value of this put option (hint: divide the 1 year period into two 6-month intervals; thus $\delta t = 0.5$).

   SOLUTION: This requires that we first determine the tree of stock prices:

$t = 0$	1	2
		€62.25
	€49.90	
€40.00		€37.55
	€30.10	
		€22.65

   The put option prices at $t = 2$ are found by computing $[\max(40 - S_2), 0]$; thus there are three possible payoffs on the put: [p_{uu}= €0, p_{ud}= €2.45, and p_{dd}= €17.35]. Note that the risk neutral probability of an up move is given by $q = \frac{(1 + r\delta t) - d}{u - d} =$ .5505, and $PVDF = 1/1.025 = .9756$. Therefore, $p_u = \frac{1}{1 + r\delta t}[qp_{uu} + (1 - q)p_{ud}]$ $= \frac{1}{1.025}[.5505(0) + (.4495)2.45] =$ €1.07, and $p_d = \frac{1}{1 + r\delta t}[qp_{ud} + (1 - q)p_{dd}] = \frac{1}{1.025}[.5505(2.45) + (.4495)17.32] =$ €8.92. Thus, the current market value of this put option is $p = \frac{1}{1 + r\delta t}[qp_u + (1 - q)p_d] = \frac{1}{1.025}[.5505(1.07) + (.4495)8.92] =$ €4.49.

2. (8 points) What is the current market value for an otherwise identical European call option (i.e., like the put option, this call option also matures in 1 year and has an exercise price of €40).

   SOLUTION: The simplest way to solve for the call option is to use the put-call parity theorem; i.e., $c + \frac{X}{(1 + r\delta t)^2} = p + S$; therefore, $c = p + S - \frac{X}{(1 + r\delta t)^2} =$ €4.49 + €40 - €38.07 = €6.42.

   Alternatively, one can calculate the value of a European call option in a manner that is similar to the way we found the European put option price. Note that call option prices at $t = 2$ are found by computing $[\max(S_2 - 40), 0]$; thus there are three possible payoffs on the call: $[c_{uu}= €22.25,\ c_{ud}= €0, \text{ and } c_{dd}= €0]$. Since the call only pays off after two up moves, the current value is given by the following equation:

$$c = \frac{q^2 c_{uu}}{(1+r\delta t)^2} = \frac{.5505^2(22.25)}{1.025^2} = €6.42.$$

3. (8 points) Now suppose that both of these options have *stochastic* exercise prices; specifically, the exercise price will be remain at €40 so long as the underlying stock ends up being worth less than €40 one year from now; otherwise, the exercise price for both options will be reset at €50. Given this information, recalculate the price of the put option. Does a stochastic exercise price affect the value of the put option? Why or why not?

   SOLUTION: The price of the put option is the same as before, because this change does not affect the put payoff after two timesteps; note that as before, $[p_{uu}= €0$, $p_{ud}= €2.45$, and $p_{dd}= €17.35]$. The only effect that a stochastic exercise price has is to make the put €10 *less* out of the money (but still, it remains out of the money) after two consecutive up moves in the stock. In the other two states, the exercise price stays the same as before. Consequently, the price of the put remains at €4.49.

4. (8 points) Given the information provided in part C, recalculate the price of the call option. Does a stochastic exercise price affect the value of the call option? Why or why not?

   SOLUTION: This change in the exercise price reduces the value of the call option by reducing the c_{uu} payoff from €22.25 to €12.25 (note that $c_{ud}= c_{dd}= €0$ as before). Thus,

$$c = \frac{q^2 c_{uu}}{(1+r\delta t)^2} = \frac{.5505^2(12.25)}{1.025^2} = €3.53.$$

   Also note that the put-call parity equation $c + \frac{X}{(1+r\delta t)^2} = p + S$ no longer applies because X is stochastic, and the put-call parity equation treats X as a fixed value.

Problem #3 (33 points)

Currently, a share of RWE sells for €25. The annualized volatility (σ) for this stock is 60 percent. Currently, the annualized risk free interest rate is 5%.

1. (11 points) Calculate the value of a European put option on RWE with an exercise price of €20 and an expiration date of 1 year from today.

   SOLUTION: Calculation of put value:

$$-d_1 = -\frac{\ln(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = -\frac{\ln(25/20) + (.05 + .5(.36))}{.60} = -0.7552.$$

Therefore, -d_2= -d_1 + $\sigma\sqrt{T}$= -0.7552 + .60 = -0.1552. Consequently, N(-d_1) = 22.51%, $N(d_2)$ = 43.83%, and the value of the put option is:

$p = e^{-rT}KN(-d_2) - SN(-d_1)$= $e^{-.05}$(€20)(.4383) - €25(.2251) =€2.71.

Applying the put-call parity theorem, $c = p + S - Ke^{-rT}$= €2.71 + €25 - $e^{-.05}$(€20) = €8.69.

2. (11 points) Suppose that a European call option on RWE with an exercise price of €20 and an expiration date of 1 year from today is priced at €5. Is this a fair price for this call option? If not, describe a riskless arbitrage strategy that can be implemented to take advantage of the mispricing. Also, calculate the profit you would receive from implementing such a strategy.

   SOLUTION: We need to determine the fair market value of the call option. Since we already know the fair market value of an otherwise identical put option, we can infer the fair market value of the call option by applying the put-call parity theorem; i.e., $c = p + S - Ke^{-rT}$= €2.71 + €25 - $e^{-.05}$(€20) = €8.69. Since the call is worth €8.69 but it is only selling for €5, it is clearly *undervalued.* We can therefore implement the following riskless arbitrage strategy to take advantage of this mispricing:

   - Purchase a portfolio consisting of the call and the present value of the exercise price: total investment = -(€5 + €19.02) = =-€24.02.
   - Fund this purchase by selling a put and short selling RWE stock: total inflow = €2.71 + €25 = €27.21. This results in a net cash inflow of €3.19.

   Since you are short in the put and stock and long in the call, irrespective of the value of the stock you will end up buying it for €20, which will allow you to cover your short position in RWE. Therefore the net profit from this riskless arbitrage strategy is €3.19.

3. (11 points) As part of a private financing deal, RWE management has decided to issue warrants on RWE stock with an exercise price of €30 and an expiration date of 2 years from today (note: warrants are privately negotiated (i.e., non-exchange traded) European call options with maturities exceeding 1 year). What is the fair market value for these warrants? Explain why there is a difference in price between the two-year warrants and the 1-year call options.

   SOLUTION: The fair market value for a warrant on RWE with an exercise price of €30 and an expiration date of 2 years from today can be calculated by applying the Black-Scholes option pricing formula; specifically, we must first calculate d_1 and d_2, then $N(d_1)$ and $N(d_2)$, and then combining these probability measures with the current value of stock and the present value of the exercise price:

$$d_1 = \frac{\ln(S/K) + (r + .5\sigma^2)T}{\sigma\sqrt{T}} = \frac{\ln(25/30) + (.05 + .5(.36))2}{.6\sqrt{2}} = 0.3272.$$

   Therefore, d_2= d_1 - $\sigma\sqrt{T}$= $0.3272 - .6\sqrt{2}$ = -0.5213. Consequently, $N(d_1)$ =62.83%; $N(d_2)$ = 30.11%, and the value of the warrant is:

$$C_0 = S_0N(d_1) - e^{-rT}KN(d_2) = €25(62.83\%) - e^{-.05(2)}(€30)(30.11\%) = €7.53.$$

Consequently, the 2-year warrants are less valuable than the 1-year call options. This is due to two countervailing effects. Everything else the same, a longer time to expiration increases option value. However, the warrant is out of the money, whereas the call is in the money due to the differences in the exercise prices of these instruments. This difference makes the warrants less valuable. In this particular problem, the negative valuation consequence of a higher exercise price on the warrant more than compensates for the positive valuation consequence of a longer time to expiration.